Received: *May 15, 2000*, Accepted: *May 31, 2000*

A stable non-BPS configuration from intersecting branes and antibranes

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Abstract: We describe a tachyon-free stable non-BPS brane configuration in type-IIA string theory. The configuration is an elliptic model involving rotated NS5-branes, D4-branes and anti-D4 branes, and is dual to a fractional brane-antibrane pair placed at a conifold singularity. This configuration exhibits an interesting behaviour as we vary the radius of the compact direction. Below a critical radius the D4 and anti-D4 branes are aligned, but as the radius increases above the critical value the potential between them develops a minimum away from zero. This signals a phase transition to a configuration with finitely separated branes.

Keywords: Superstrings and Heterotic Strings, D-branes.

JHEP06(2000)001

Contents

1. Introduction and review

Much has been learned in recent times about the physics of brane-antibrane pairs and non-BPS branes in superstring theory [1]–[32]. Parallel, infinitely extended pairs attract each other, and can annihilate into the vacuum by a process of tachyon condensation into a constant minimum. An analogous decay process takes place for single or multiple non-BPS branes. Condensation of the tachyon as a kink, vortex or more general soliton is associated to brane-antibrane annihilation into branes of lower dimension.

The above considerations have been extended to backgrounds with lower supersymmetry (orientifolds, orbifolds and smooth Calabi-Yau manifolds), where one finds new phenomena including the existence of stable, non-BPS branes. As parameters of the background are varied, there can also be phase transitions between qualitatively different configurations. The reader may consult the reviews in refs. [33]–[36].

A different direction, explored in ref. [37], is to consider non-BPS configurations of intersecting branes and antibranes in the fully supersymmetric type-II spacetime background. Here one encounters novel phenomena including both attractive and repulsive interactions among branes and antibranes. Such configurations could be useful to study non-supersymmetric field theories, and also to understand better the basic underlying structure of superstring theory.

In the present note we examine a variant of a configuration of "adjacent brane-antibrane pairs" that was discussed in ref. [37]. Let us describe the original configuration. In type-IIA theory we start with a pair of parallel NS5-branes extended along x^1, x^2, x^3, x^4, x^5 and separated along x^6. The x^6 direction is compact, with circumference $2L$. Now stretch a D4-brane along x^6 from the first NS5-brane to the second, and a $\overline{\mathrm{D}}$4-brane along x^6 from the second NS5-brane to the first (figure 1).

In ref. [37] it was argued that the D4-brane and $\overline{\mathrm{D}}4$-brane exert a net repulsive force on each other, with the result that the configuration is unstable. From the point of view of the field theory on the NS5 world volume, this repulsion is essentially due to the fact that the D4 and $\overline{\mathrm{D}}4$ end on the NS5-brane from opposite sides, and their ends are charged 3-branes in the NS5 world volume. If we dimensionally reduce everything over these three directions, then the ends become vortices living on the reduced NS5 world volume. These vortices carry the same charge under the gauge field, hence they repel, and since the configuration is non supersymmetric there is no reason to expect that this repulsion is cancelled by exchange of other massless fields. Because the NS5-branes are parallel, the repelling D4- and $\overline{\mathrm{D}}4$-branes can run away from each other to infinity.

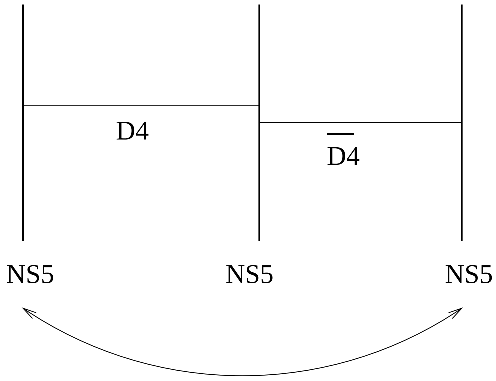

Figure 1: Adjacent D4 and $\overline{\mathrm{D}}4$ between parallel branes.

This instability can also be understood in the T-dual picture, where the D4- and $\overline{\mathrm{D}}4$-branes are actually two types of fractional branes (denoted 1_f and $\overline{1}'_f$, respectively in [37]) at a $\mathbb{Z}_2$ ALE singularity. These two fractional branes repel, as they each carry a full unit of twisted RR charge. There is also an attraction due to the fractional untwisted RR charge, but an explicit computation of the amplitude using orbifold techniques [37] reveals that the repulsive force dominates.

The variant of this configuration that we will describe in the next section involves rotating the NS5-branes. This converts the T-dual ALE space into a conifold [38, 39], hence this brane construction is now T-dual to fractional branes at a conifold, for which we cannot use orbifold techniques to compute the force. We will analyse the model using some observations in refs. [38, 39, 40, 15, 37] and argue that this time a stable non-BPS configuration is obtained.

2. A stable configuration

Consider the following brane configuration in type IIA: an NS5-brane filling x^1, x^2, x^3, x^4, x^5 and located at $(x^8, x^9) = (0, 0)$, and another NS5-brane (denoted by NS5') filling x^1, x^2, x^3, x^8, x^9 and located at $(x^4, x^5) = (0, 0)$. The two branes are placed at the same point in the x^7 direction and are separated along the compact x^6 direction of circumference R_6.

Suspend a D4-brane between NS5-NS5' in one of the two intervals and a $\overline{\mathrm{D}}4$ in the other, so that the 4-branes extend along x^1, x^2, x^3 and x^6 (figure 2). This configuration breaks all the supersymmetries of type IIA, though each of the D4 and $\overline{\mathrm{D}}4$, together with the NS5-branes, separately preserves some supersymmetry.

Unlike the case discussed in the previous section, here the configuration of NS5-branes is no longer dual to an ALE space but rather to a conifold [38, 39]. Nevertheless, one can still argue that the stretched D4- and $\overline{\text{D}}$4-branes repel. In the case of parallel NS5-branes, the repulsion was identified as coming from like charges carried by the ends of the 4-branes. In this picture the repulsive effect is localised on each NS5-brane separately, hence introducing a relative rotation should not matter. Moreover, from the string theory point of view, the repulsion between the D4 and $\overline{\text{D}}$4 is obtained by calculating a closed-string tree amplitude (cylinder amplitude). Translated into the open-string channel, this is a one-loop open-string amplitude. This amplitude depends only on the spectrum obtained by quantizing the open strings connecting D4 and $\overline{\text{D}}$4 across any one NS5-brane. This again suggests that the force is localized near one NS5-brane at a time.[1] Using these arguments, we conclude that the D4- and $\overline{\text{D}}$4-branes in figure 2 repel each other, and that the repulsion is the same as that between adjacent D4- and $\overline{\text{D}}$4-branes when the NS5-branes are not rotated with respect to each other.

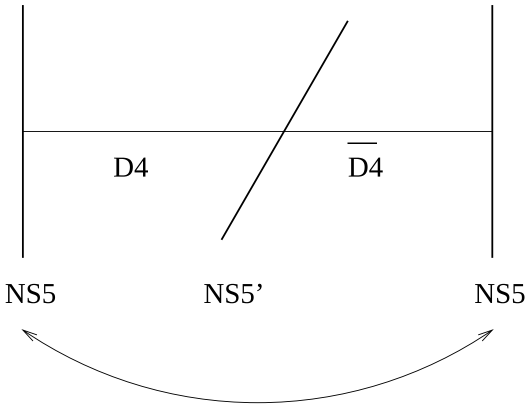

Figure 2: Adjacent D4 and $\overline{\text{D}}$4 between rotated branes.

With rotated NS5-branes, the important difference is that the D4-branes no longer have moduli to move away from each other. As they move with their ends on the NS5-branes, the D4-branes get stretched. In the process their effective 3-brane tension increases, providing a restoring force for the repelling ends of the adjacent $\text{D4} - \overline{\text{D}}4$. Thus one can expect a configuration in which the repulsive force and the restoring force due to the increased tension of the adjacent $\text{D4} - \overline{\text{D}}4$ pair exactly cancel, giving rise to a configuration that is stable at least under small perturbations.

In fact, as we now show explicitly, such a stable configuration exists for some range of values of the circumference R_6. For simplicity, let us assume that the NS5- and NS5'-branes are located at diametrically opposite points on the compact x^6 direction, with the separation between them being $L = \frac{1}{2} R_6$. With this, and the fact that the branes are rotated at 90 degrees to each other, there is a high degree of symmetry in the problem. If we let r be the displacement of the end of the D4-brane from the origin in the x^4 (or x^5) direction, then the $\overline{\text{D}}$4-brane will also be displaced by an equal amount r, and the displacement of the other ends of the 4-branes along x^8 (or x^9) will also be r (figure 3).

[1]This feature of open strings across NS5-branes was exploited in refs. [38, 39] to obtain the spectrum of the gauge theory living on related (BPS) brane configurations.

With the above data, the net tension of the stretched D4($\overline{\rm D}4$) is $\mathcal{V}\, T_4\sqrt{L^2+2r^2}$ where $\mathcal{V}$ is the (infinite) 3-volume of the (x^1, x^2, x^3) directions and $T_4 = \frac{1}{g_s(2\pi)^4}$ is the tension of a BPS D4-brane. The contribution from the repulsion between the ends of D4- and $\overline{\rm D}4$-branes on an NS5-brane to the energy of the system is given by [37]:

$$\frac{\mathcal{V}}{16(2\pi)^4}\int_0^\infty \frac{dt}{t^3}\, e^{-2X^2t/\pi}\, \mathcal{F}(q)\,. \tag{2.1}$$

Here $X = 2r$ is the separation of D4 and $\overline{\rm D}4$ along the NS5-brane, and $q = \exp(-\pi t)$. The function $\mathcal{F}(q)$ is given by

$$\mathcal{F}(q) = \frac{f_4(q)^8}{f_1(q)^8}\left[1 - 4\,\frac{f_1(q)^4\, f_3(q)^4}{f_2(q)^4\, f_4(q)^4}\right], \tag{2.2}$$

where the $f_i(q)$ are defined as:

$$\begin{aligned} f_1(q) &= q^{1/12}\prod_{n=1}^{\infty}\left(1-q^{2n}\right), & f_2(q) &= \sqrt{2}\, q^{1/12}\prod_{n=1}^{\infty}\left(1+q^{2n}\right), \\ f_3(q) &= q^{-1/24}\prod_{n=1}^{\infty}\left(1+q^{2n-1}\right), & f_4(q) &= q^{-1/24}\prod_{n=1}^{\infty}\left(1-q^{2n-1}\right). \end{aligned} \tag{2.3}$$

Dropping the common factor $\mathcal{V}$, the total potential energy of the system of branes in figure 3 is

$$\begin{aligned} V(r) &= \frac{1}{g_s(2\pi)^4}\sqrt{L^2+2r^2} - \frac{1}{16(2\pi)^4}\times \\ &\quad\times\int_0^\infty \frac{dt}{t^3}\, e^{-8r^2t/\pi}\,\mathcal{F}(q) \\ &= V^{(1)}(r) + V^{(2)}(r)\,. \end{aligned} \tag{2.4}$$

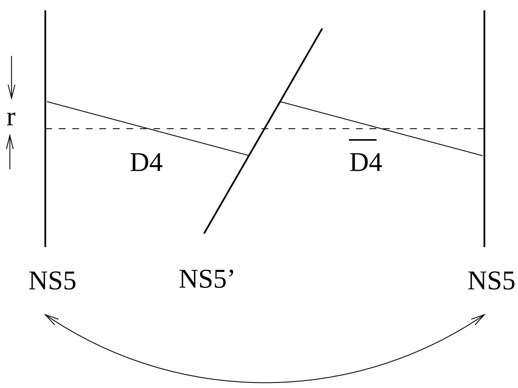

Figure 3: Equilibrium configuration after displacement of 4-branes.

This expression can be minimized to get the condition for the equilibrium value for r.

The second term in eq. (2.4) is somewhat complicated. So we first analyze it in two different limits and extract some physical information. In the $r \to \infty$ limit, the most significant contribution comes from the $t \to 0$ behaviour of $\mathcal{F}(q)$.

$$\mathcal{F}\left(e^{-\pi t}\right) \longrightarrow -16\, t^2 \quad \text{as } t \to 0\,. \tag{2.5}$$

Therefore we have

$$V^{(2)}(r) = -\frac{1}{16(2\pi)^4}\int_0^\infty \frac{dt}{t^3}\, e^{-8r^2t/\pi}\left(-16\, t^2\right), \quad r \gg 1 \tag{2.6}$$

(we are measuring distances in units of $\sqrt{\alpha'}$). This integral diverges logarithmically because of the behaviour of the integrand in the $t \to 0$ limit. To extract the behaviour of this quantity as a function of r, let us regulate it by putting a cut-off ϵ for the lower limit of the integration variable t, and then take $\epsilon \to 0$. Thus eq. (2.6) becomes

$$V^{(2)}(r) = \frac{1}{(2\pi)^4}\left[-\gamma - \log\left(\frac{8r^2\epsilon}{\pi}\right) - \sum_{n=1}^{\infty}\frac{(-1)^n(8\,r^2\epsilon/\pi)^n}{n.n!}\right], \quad r \gg 1\,, \tag{2.7}$$

where γ is the Euler constant. In the limit $\epsilon \to 0$ the third term vanishes and we are left with a potential of the form:

$$V^{(2)}(r) = A - B\,\log(r)\,, \quad r \gg 1\,, \tag{2.8}$$

where A is an infinite constant and $B = 2/(2\pi)^4$. From this the contribution of the second term in eq. (2.4) to the force between the two D4-branes, given by $-dV^{(2)}/dr$, is

$$F^{(2)}(r) = \frac{B}{r}\,, \quad r \gg 1\,. \tag{2.9}$$

Thus in the large-separation limit this contribution to the force between two 4-brane segments is repulsive, as expected.

Now let us look at the behaviour of this contribution for small values of r. In this limit we can expand the exponential in eq. (2.4) in powers of r to get

$$V^{(2)}(r) = C - D\,r^2, \quad r \ll 1\,, \tag{2.10}$$

where

$$C = \frac{1}{8(2\pi)^4}\int_0^{\infty}\frac{dt}{t^3}\,\mathcal{F}(q)\,, \qquad D = -\frac{1}{(2\pi)^5}\int_0^{\infty}\frac{dt}{t^2}\,\mathcal{F}(q)\,. \tag{2.11}$$

Notice that C is a divergent integral whereas D is convergent. D is also positive because $\mathcal{F}(q)$ is negative all through the range of integration. From eq. (2.10) the small-r behaviour of the force turns out to be

$$F^{(2)}(r) = 2\,D\,r\,, \quad r \ll 1 \tag{2.12}$$

which is also repulsive, as expected. From eq. (2.4), the restoring force is:

$$F^{(1)}(r) = -\frac{dV^{(1)}}{dr} = -\frac{1}{g_s(2\pi)^4}\,\frac{2r}{\sqrt{2L^2 + (2r)^2}} \tag{2.13}$$

which is attractive as explained above. The strength of attraction depends on the value of L, related to the size of the compact x^6 direction.

We want to know whether there is a stable minimum of the total potential, and under what conditions this minimum is attained away from $r = 0$. In order to argue for the presence of a stable minimum at non-zero separation of the brane-antibrane

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pair, it is sufficient to show that the potential has an unstable turning point at the origin. Combined with the attractive behaviour for large r, this suffices to show that the potential develops a stable minimum somewhere in between.

From eqs. (2.4) and (2.10), we have for $r \ll 1$,

$$V(r) \simeq \frac{1}{g_s(2\pi)^4}\frac{r^2}{L} - Dr^2 \tag{2.14}$$

upto additive constants. Here, D is the positive constant given in eq. (2.11). It follows that V has a turning point at the origin that is unstable (tachyonic) when L is greater than a critical value L_c, namely:

$$L > L_c = \frac{1}{g_s(2\pi)^4 D}\,. \tag{2.15}$$

The function $\mathcal{F}(q)$ defined in eq. (2.2) tends to the constant value -8 as $t \to \infty$. Hence an estimate for D can be made by approximating $\mathcal{F}$ in the integrand in eq. (2.11) by $-16\,t^2$ for $0 < t < 1/\sqrt{2}$ and -8 for $1/\sqrt{2} < t < \infty$. With this, we find

$$(2\pi)^4 D \sim \frac{16\sqrt{2}}{2\,\pi} \sim 3.60 \tag{2.16}$$

so the phase transition takes place at $L_c \sim 0.28\, g_s^{-1}$.

We expect that there will be no loop corrections to the restoring potential $V^{(1)}$, as this depends only on the D-brane tension which is unrenormalized. The repulsive potential $V^{(2)}$ will, on the other hand, receive loop corrections, but they are independent of L, and can be expected to be small for sufficiently small g_s. Hence we do not expect stringy corrections to invalidate the conclusions of this section.

Some further analysis of this potential can be found in the appendix.

3. The T-dual configuration

It has been argued that the elliptic configuration involving rotated NS5-branes is T-dual to the conifold geometry [38, 39]. Above we have studied an adjacent $\mathrm{D4}-\overline{\mathrm{D}}4$ pair intersecting with this elliptic configuration. Hence one may ask what is the precise configuration, involving a suitable $\mathrm{D3}-\overline{\mathrm{D}}3$ pair at a conifold, obtained by T-duality along the compact x^6 direction. Such a configuration should describe a stable non-BPS system exhibiting a phase transition.

As discussed above in the introduction, there is a simpler situation where the analogous T-duality relation holds: the elliptic model of two parallel NS5-branes [41], which is T-dual [42] to a $\mathbb{Z}_2$ ALE geometry. An adjacent $\mathrm{D4}-\overline{\mathrm{D}}4$ pair in this geometry is dual to a particular pair of fractional branes at an ALE space [37]. In this system, the adjacent brane-antibrane pair can separate along the (x^4, x^5) directions, which lie within the bounding NS5-branes. In the T-dual picture the fractional branes live in the 5-plane transverse to the ALE space, and due to their mutual repulsion, they separate along the same directions (x^4, x^5), that are transverse both to their own worldvolume and to the ALE space. In this discussion, one gets a satisfactory physical picture without having to take into account the back-reaction of the D-branes on the NS5-branes, or on the geometry.

For rotated NS5-branes the situation is somewhat different. On the one hand, the model of a wrapped D4-brane intersecting with rotated NS5-branes is fairly similar to the one with parallel NS5-branes: locally there is always a D4-brane ending on a codimension-2 locus inside an NS5-brane. On the other hand, in the T-dual conifold geometry we know [43] that D3-branes completely smoothen out the conifold singularity: the near-horizon geometry becomes $AdS_5 \times \mathbb{T}_{1,1}$. Thus in the latter picture the back reaction of the branes on the geometry is qualitatively very important. This can be traced to the fact that the branes completely fill the space transverse to the singularity.

The stable non-BPS configuration discussed in the previous section is an example of this type. While it can be visualised explicitly in the brane-construction picture, it is not so easy to describe in terms of branes at a conifold. For very weak string coupling (and fixed L) the problem is somewhat easier, since in this case the $\mathrm{D4}-\overline{\mathrm{D}}4$ pair is aligned. The T-dual configuration will consist of a pair of fractional branes (more precisely, the first fractional part of a BPS brane, along with the second fractional part of a BPS antibrane) at the conifold. As for the case in ref. [43], here too we expect the conifold geometry to be smoothed out near the origin. As the string coupling increases, a phase transition takes place and the $\mathrm{D4} - \overline{\mathrm{D}}4$ separates, as discussed above. In this case the T-dual configuration is harder to visualise. The fractional pair cannot separate in any direction transverse to the conifold, so it must be thought of as separating within the conifold directions. Asymptotically this should look like a $\mathrm{D3} - \overline{\mathrm{D}}3$ pair at separated locations away from the conifold singularity. But close to the origin the behaviour could be more complicated, with the conifold geometry being replaced by a more non-trivial one.

Both these situations should be amenable to study as supergravity solutions. With N_1 branes in the first segment and N_2 antibranes in the second, and for sufficiently large N_1 and N_2, there should be a trustworthy non-supersymmetric supergravity solution dual to the RG flow of a non-supersymmetric $\mathrm{SU}(N_1)\times\mathrm{SU}(N_2)$ gauge theory. This situation is very similar to the one recently considered in ref. [44]–[47], except that these authors considered supersymmetric configurations with full branes and fractional branes. If each full brane is replaced by a fractional brane-antibrane pair (in the sense discussed above) then our desired configuration is obtained. Since in this process supersymmetry is completely broken, it remains to be seen whether an explicit solution can be found. This should be a fascinating direction to explore, as one would hope to see our phase transition as an instability of the supergravity solution when some parameter is varied.

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4. Summary and discussion

We have exhibited a configuration of adjacent D4- and $\overline{\text{D}}$4-branes in type-IIA string theory, suspended between relatively rotated NS5-branes, which corresponds to a stable non-BPS state. A crucial assumption was that the force between adjacent brane-antibrane pairs can be estimated using a "locality" property, according to which it originates from the repulsion between the ends of these 4-branes in the NS5-brane worldvolumes. This repulsion can in turn be computed using standard orbifold techniques, valid for the model with parallel NS5-branes which is dual to a $\mathbb{Z}_2$ ALE singularity.

While we do not know at present how to estimate the validity of this assumption, it is encouraging that it gives a definite and physically reasonable answer. However, there are some grounds for caution.[2] When the two NS5-branes in the ALE case are far apart (distance much greater than unity in our units), the T-dual theory has a pair of closely spaced ALE spaces. In this case it is not justified to concentrate only on one of the ALE spaces, and phenomena involving branes wrapped on the 2-cycles of both of them can occur [48, 49]. Hence the region where the ALE calculation should be most reliable is $L \ll 1$. On the other hand, this calculation was an input in finding the phase transition for the case of rotated NS5-branes, which as we have seen occurs (at weak coupling) for L_c rather close to 1. Therefore one should perhaps be cautious in taking the numbers in the previous section too seriously.

As we have indicated in the previous section, a supergravity calculation might be one route to provide an independent check of our conclusions.

The $(3+1)$-dimensional field theory on the common worldvolume in our brane construction will be a non supersymmetric, tachyon-free theory. Because the model is elliptic, it should flow to a CFT. One can generalise the model to include N_1 D4-branes in one segment and N_2 $\overline{\text{D}}$4-branes in the other segment. For $N_1 = N_2 = N$ this will again flow to a CFT. Its large-N limit should be interesting.

There are various other generalisations of our model which we have not discussed here but should be quite straightforward to analyse. This includes choosing the relative rotation of the NS5-branes to lie somewhere between 0 and $\pi/2$, introducing more NS5-branes rotated at various angles [38], and varying the spacing between the NS5-branes. One can also study non-elliptic models and incorporate semi-infinite D4-branes and $\overline{\text{D}}$4-branes.

Above the critical radius, our model provides a situation where a brane and an antibrane are at a finite separation that is calculable in terms of various parameters including the string coupling and the radius of a compact direction. Such configurations might perhaps be useful to construct novel "brane-world" type models.

[2]We thank Oren Bergman for a discussion on this issue.

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Acknowledgments

We would like to thank Atish Dabholkar and Sandip Trivedi for helpful discussions. We are particularly grateful to Sandip Trivedi for a careful reading of the manuscript.

A. Detailed analysis of the potential

The numerical value of L_c below eq. (2.16) is only approximate, since we have made a crude estimate for the integral in eq. (2.11). An improvement on this estimate can be made by taking

$$\begin{aligned} \mathcal{F}(q) &\sim -16\,t^2 + 16\,t^4\,, \quad t \text{ small}\,, \\ \mathcal{F}(q) &\sim -8 + 45\,q\,, \quad t \text{ large} \end{aligned} \tag{A.1}$$

and then finding an intermediate value of t at which these two functions match. We find that t is shifted from $1/\sqrt{2}$ to ~ 0.76, and the value of $(2\pi)^4 D$ decreases from 3.60 to 3.02. As a result, L_c moves up to about $0.33\,g_s^{-1}$, an increase of 18%. This suggests that at least the order of magnitude of L_c has been correctly estimated.

One may wonder if the potential has a unique minimum away from 0 for $L > L_c$ or whether there are several minima, some of them metastable. For this, it is convenient to make the same approximation above, but not just for the $r \ll 1$ behaviour. We take the term $V^{(2)}(r)$ in eq. (2.4) and write it as follows:

$$\begin{aligned} V^{(2)}(r) &= -\frac{1}{16(2\pi)^4}\int_0^\infty \frac{dt}{t^3}\, e^{-8r^2t/\pi}\, \mathcal{F}(q) \\ &\sim -\frac{1}{16(2\pi)^4}\int_0^{1/\sqrt{2}} \frac{dt}{t^3}\, e^{-8r^2t/\pi}\, \left(-16\,t^2\right) - \\ &\quad -\frac{1}{16(2\,\pi)^4}\int_{1/\sqrt{2}}^\infty \frac{dt}{t^3}\, e^{-8r^2t/\pi}(-8)\,. \end{aligned} \tag{A.2}$$

The integrals can now be evaluated. It is convenient to rescale the distance by defining $y = \frac{2^{5/4}}{\sqrt{\pi}}\,r$, then we find:

$$V^{(2)} \sim \frac{1}{(2\pi)^4}\left(\frac{1}{2}\left(1-y^2\right)e^{-y^2} + \left(1-\frac{y^4}{2}\right)\mathrm{Ei}\left(-y^2\right) - 2\log y - \frac{1}{2} - \gamma\right). \tag{A.3}$$

Here, Ei is the exponential-integral function and γ is the Euler constant. We have dropped an infinite constant associated to the logarithmic term in the potential, and subtracted a finite constant $-\frac{1}{2} - \gamma$ to make the potential vanish at the origin.

Now we add the first term in eq. (2.4), which we write:

$$V^{(1)} = \frac{1}{g_s(2\pi)^4}\sqrt{L^2+2r^2} - L = \frac{\sqrt{\pi}}{2^{3/4}\,g_s(2\pi)^4}\left(\sqrt{\widetilde{L}^2+y^2} - \widetilde{L}\right), \tag{A.4}$$

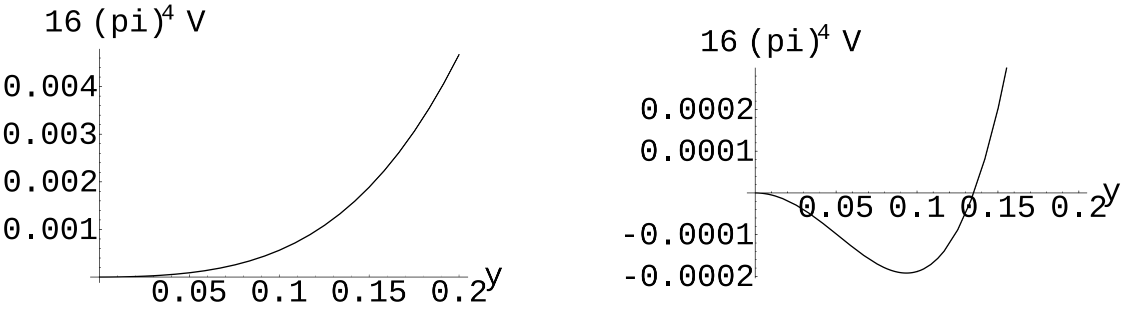

Figure 4: Potential for $\widetilde{L} = 2.6, 2.7$.

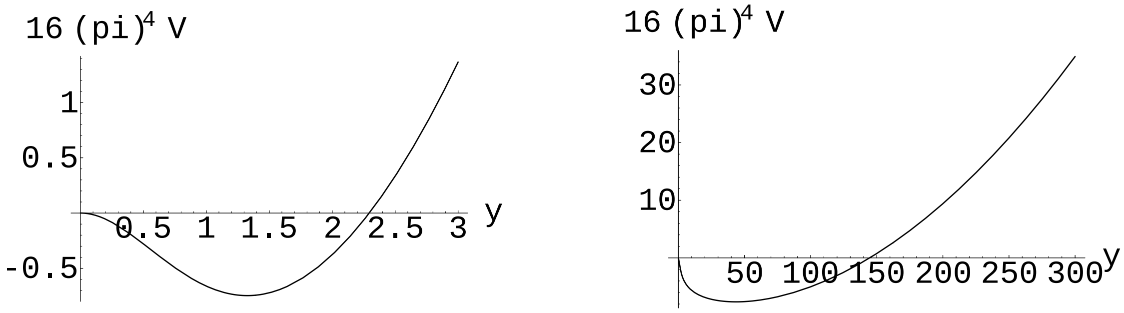

Figure 5: Potential for $\widetilde{L} = 10, 10000$.

where $\widetilde{L} = \frac{2^{3/4}}{\sqrt{\pi}} L$, and again a constant has been subtracted to make the function vanish at the origin. It is now straightforward to plot $V^{(1)}(y) + V^{(2)}(y)$ for different values of $\widetilde{L}$ (figures 4 and 5). In these plots we have set $g_s = 0.1$. For this value of g_s, the phase transition is at $\widetilde{L}_c \sim 2.65$. We see that, at least for the $\widetilde{L}$ values in the plots, there seems to be a unique and non-zero minimum when $\widetilde{L} > \widetilde{L}_c$.

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